Equivalent resistance

Whenever several resistors are combined in a circuit, they can be replaced by an **equivalent resistance** (or total resistance).

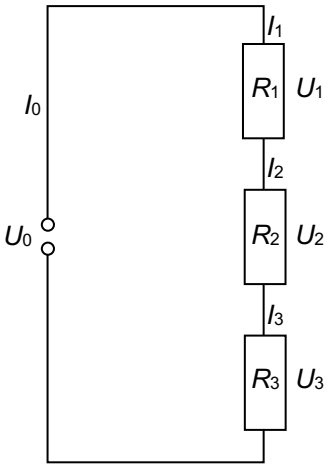
Task: Find an equation for calculating the equivalent resistance of a circuit from the individual resistances.

1. Equivalent resistance of resistors in series

In a series circuit the following rules apply:

a) The current is the same through all resistors

b) The total voltage applied across all the resistors is divided, thus it equals the sum of the separate voltages



Written as an equation:

a) I_0 = ...

b) U_0 = ...

Strategy:

1. The equivalent resistance R_{eq} is calculated from the total voltage U_0 and the total current I_0:

   R_{eq} =

2. The total voltage U_0 across all resistors is the sum of the individual voltages U_1, U_2 and U_3. Substitute U_0 in the above equation with the right side of equation b). Write the fraction as the sum of three individual fractions:

   R_{eq} = =

3. The individual resistances are calculated from the voltage across each individual resistance and the current through it: $R_1 = \frac{U_1}{I_1}$, $R_2 = \frac{U_2}{I_2}$, etc. Substitute I_1, I_2 and I_3 with I_0 (see a):

   R_1 = R_2 = R_3 =

4. Compare 3. and 2. Find an equation for calculating the equivalent resistance *solely* from the individual resistances:

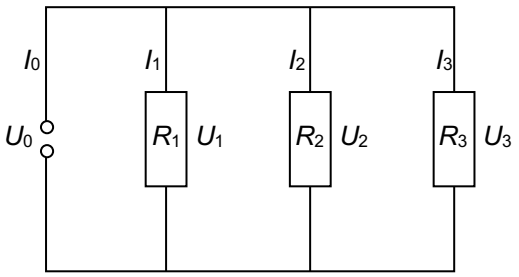
   R_{eq} =

2. Equivalent resistance of resistors in parallel

<u>In a parallel circuit the following rules apply:</u>

a) The voltage is the same across all resistors

b) The total current splits to flow through each resistor, thus the total current equals the sum of the separate currents



Written as an equation:

a) U_0 =.....

b) I_0 = ...

<u>Strategy:</u>

1. The equivalent resistance R_{eq} is calculated from the total voltage U_0 and the total current I_0:

   R_{eq} =

2. The total current I_0 through all resistors is the sum of the individual currents I_1, I_2 and I_3. Substitute I_0 in the above equation with the right side of equation b):

   R_{eq} =

3. Write the reciprocal of equation 2. Replace the fraction with the sum of three individual fractions:

   $\frac{1}{R_{eq}}$ = =

4. The individual resistances are calculated from the voltage across each individual resistance and the current through it: $R_1 = \frac{U_1}{I_1}$, $R_2 = \frac{U_2}{I_2}$, etc. Replace U_1, U_2 and U_3 with U_0 (see a):

   R_1 = R_2 = R_3 =

5. Write the reciprocals:

   $\frac{1}{R_1}$ = $\frac{1}{R_2}$ = $\frac{1}{R_3}$ =

6. Compare 5. and 3. Find an equation for calculating the equivalent resistance *solely* from the individual resistances:

   $\frac{1}{R_{eq}}$ =