Equal laws of nature for an apple and the moon?
Newton's universal Law of Gravitation

Introduction: What idea of Newton's was completely new at the time?

Follow Newton's thoughts to find his law of gravitation:

1. Draw the situation:

An apple at the Earth's surface experiences the acceleration g_{Earth} towards the center of the Earth.
The apple's distance from the center of the Earth is r_{Earth} = 6'370 km (radius of the Earth).

The Moon experiences the centripetal acceleration a_c towards the center of the Earth. Its period (time for one revolution) is T = 27.322 d.
The centers of the Moon and the Earth are $r_{\text{Earth–Moon}}$ = 384'400 km apart.

2. Compare the following values:

a) What is an apple's acceleration of free fall g_{Earth} on the Earth's surface?

g_{Earth} =

b) Calculate the Moon's centripetal acceleration a_c from its period T (time for one revolution) and the distance between the centers of the Moon and the Earth $r_{\text{Earth–Moon}}$.

a_c =

c) Compare the apple's distance from the Earth's center r_{Earth} to the distance of the Moon's center from the Earth's center $r_{\text{Earth–Moon}}$ (round to two digits).

ratio: $\frac{r_{\text{Earth–Moon}}}{r_{\text{Earth}}}$ =

d) Compare the apple's acceleration on the Earth's surface g_{Earth} to the Moon's acceleration a_c (round to two digits).

ratio $\frac{g_{\text{Earth}}}{a_c}$ =

e) Complete the following sentences:

The Moon's acceleration is .... (*greater* / *smaller*) than the apple's acceleration.

The further one moves away from the center of the Earth, the .... (*greater* / *smaller*) the acceleration is.

The Moon's distance from the Earth's center is .... times greater than the apple's distance from the Earth's center.

The apple's acceleration is .... times greater than the Moon's acceleration.

The Moon's acceleration is .... of the apple's acceleration.

f) How does the acceleration depend on the distance? Which one of the following options holds?

$a_Z \sim r$ — the acceleration is proportional to the distance, i.e. at twice the distance we have twice the acceleration

$a_Z \sim \frac{1}{r}$ — the acceleration is inversely proportional to the distance, i.e. at twice the distance we have half the acceleration

$a_Z \sim r^2$ — the acceleration is proportional to the square of the distance, i.e. at twice the distance we have four times of the acceleration

$a_Z \sim \frac{1}{r^2}$ — the acceleration is inversely proportional to the square of the distance, i.e. at twice the distance we have a quarter of the acceleration

3. Let's think about the forces acting between the two objects

- ☞ According to Newton's second law, the force required to accelerate an object is proportional to is mass, i.e. the force of the Earth acting on the Moon is $F_{\text{Earth}} = m_{\text{Moon}} \cdot a_{c(\text{Moon})}$
- ☞ According to Newton's third law, the Moon also exerts a force (of the same magnitude, but opposite direction) on the Earth, accelerating it towards the Moon $F_{\text{Moon}} = m_{\text{Earth}} \cdot a_{\text{Earth}}$
- ☞ Thus, the attractive force between the Moon and the Earth is proportional to the product of their masses and inversely proportional to the square of the distance between their centers

If you've been able to follow all of this, you may call yourself NEWTON for the rest of the day!

This is Newton's **Universal Law of Gravitation**:

All masses attract each other.

The attractive force between two objects of masses m_1 and m_2 with their centers spaced apart by the distance r is given by

$F =$

The constant of proportionaliy G is the universal gravitational constant $G = 6.67 \cdot 10^{-11}\ \frac{\text{N·m}^2}{\text{kg}^2}$

G was measured by Henry Cavendish in 1798.